KY Course 44: Mathematics for Property Valuation
First Day Reading Assignment
Rev. 2005

INTRODUCTION

The purpose of this course is to provide the student with a comprehensive review of basic mathematical functions. These skills are necessary for assessors, deputies and field appraisers to properly carry out the assessment function. This course develops a systematic, logical review of key processes which will be used on a daily basis in any assessment office.

Presented in an expanded outline form, the course will provide a step by step explanation of each concept or process, followed by an example using actual numbers and loads of practical exercises with their solutions for immediate feedback.

You should make every effort to solve the problems without looking back into the course materials. The work should also be done without the use of a calculator in order to better retain the fundamental methods of performing basic math functions. The calculator may be used to check your work.

The course is divided into ten chapters and covers a review of the basic arithmetic functions of addition, subtraction, division and multiplication. There is a chapter on decimals, fractions and percentages. The course also includes basic algebra and geometry. Each section will have numerous problems to be solved, which will give the hands on experience necessary to retain the process. Sharpen those pencils, put away those crutches known as calculators and lets have some real fun!

CHAPTER ONE

SYMBOLS AND VOCABULARY IN MATHEMATICS

Student Objective - Students will identify and interpret the meaning of the most common symbols used in mathematics including function symbols, relationship symbols, and unknowns. Basic vocabulary will be reviewed and students will solve problems having multiple functions and negative numbers.

Student Benefits - Students will practice performing math operations which are used every day in the typical assessment office. Knowledge of proper math processes will minimize mistakes when calculating assessments for each property.

1. Symbols

- **Numerals** we recognize as 1, 2, 3, 4, etc., are symbols for numbers and words.

- Are used to shorten lengthy verbal descriptions of mathematical operations.

Operation		Example	Verbal Description
Addition	+	3 + 5	Three plus five
Subtraction	-	5 - 3	Five minus three
Division	:, /, and ÷	10/2	Ten divided by two
Multiplication	*, x, •, ( ), ab	2*3, 2•3	Two times three

- Are used to express relationships between numbers.

Relationship		Example	Verbal Description
Greater than	>	5 > 3	Five is greater than three
Less than	<	3 < 5	Three is less than five
Equal to	=	3 + 5 = 8	Three plus five equals eight
Not equal to	≠	3 ≠ 5	Three is not equal to five
Absolute Value	| |	|-6| = 6	The absolute value of negative six is equal to six
Percentage Change	Δ	5 Δ 3	On a calculator, will find the percent increase from 3 to 5

- Are used to represent miscellaneous terms and objects in mathematics

Term		**Example**	**Verbal Description**
Angle	∠	∠ABC	Angle ABC
Degrees	°	45° N	Forty five degrees north
Minutes	′	35° 12′ NW	35 degrees, 12 minutes northwest
Seconds	″	35° 12″ NW	35 degrees, 12 seconds northwest
Right Angle	∟		
Percent	%	35%	Thirty five percent
Square Root	√	√49 = 7	The square root of 49 equals seven

2. **Parenthesis**

- Used to express the multiplication function.

  Example; 3(4) Means to multiply three by four
  6(3) = 18 Six times three equals 18

- Used to indicate that an expression is to be treated as a single number.

  Example; 3(4 + 1) = 15 (4 + 1) is to be treated as 5
  3(5) = 15 Rewritten, 3 times five equals fifteen

- Brackets may be substituted for an outside parenthesis when more than one set of parenthesis is used.

  Example; 6{3 + (4 + 5)}

NOTE: Each open parenthesis or bracket must be accompanied by a closed one to ensure all functions are performed correctly.

3. **Unknowns**

- Letters of the alphabet represent quantities, factors or numbers whose value is undetermined.

  Example; N + 7 Seven is increased by an unknown number.

- The first letter of a word is generally used to represent that concept in mathematical formulae.

  Example; Area = A Length = L Width = W

- The same number or quantity is represented by the same letter in longer equations.

  Example: A + A = 8 A must equal 4, as only 4 + 4 = 8

- Different numbers or quantities must be assigned different letters if those quantities are unknown.

  Example; A + B = 8 A could equal 5 and B, 3, or any other combination of two different numbers which total 8.

4. **Performance Order**

- In order to correctly solve problems with multiple operations and parentheses, the following rules must be performed **exactly** as presented below.

Rules:

1. Perform all operations within the parenthesis first.
2. For multiple parentheses, start with the inner most set first, working out following rules 3 and 4.
3. Perform all multiplication and division operations.
4. Perform all addition and subtraction operations.

Example; Solve, 3(2 + 4) + (3 + (1 + 2))

3(2 + 4) + (3 +3)
3(6) + 6
18 + 6
24

5. **Absolute Value**

- The absolute value of a number is the positive value of the number.
- Absolute value is represented by vertical, parallel lines framing a number.

Example; $|-40| = 40$ The absolute value of negative 40 equals 40

$|40| = 40$ The absolute value of 40 equals 40

6. **Signed Numbers**

- Any number may have two (2) signs,
  - Positive, represented by a plus sign (+) or no sign at all.

    Example, +3 or 3
  - Negative, represented by a minus sign (-)

    Example, -3 or -5
  - Zero is considered neither positive or negative

a. <u>Comparing Signed Numbers</u>

- The relationship of signed numbers is best illustrated using a number line.

|_____|_____|_____|_____|_____|_____|_____|_____|_____|_____|_____|_____|

-6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6

NOTE: Numbers increase in magnitude or value by moving from the left to the right.

Example; -2 is greater than -3 but is less than -1
0 is greater than -1
4 is greater than 3 but less than 5

b. <u>Rules for Comparing Signed Numbers</u>

1. All positive numbers are greater than zero.

   Example, $+5 > 0$

2. All negative numbers are less than zero.

   Example, $-5 < 0$

3. All positive numbers are greater than negative numbers.

   Example, $+5 > -2$ or $1 > -10$

4. The greater of two positive numbers has the greater absolute value

   Example, $+8 > +5$, then $|8| > |5|$

5. The greater of two negative numbers has the smaller absolute value.

   Example, $-5 > -8$, then $|-5| < |-8|$

c. <u>Rules for Adding Signed Numbers</u>

- **Like signs** - add the absolute value of the numbers and retain the common sign.

  Example; $+7 + +3 = 10$ and $-7 + -3 = -10$

- **Unlike signs** - ignore the signs, subtract the small number from the larger number and use the sign of the larger number.

  Example; $+7 + -3 = +4$
  $-7 + +3 = -4$

- **Like numbers and Unlike signs** - the result is zero.

  Example; $-14 + +14 = 0$

d. Rules for Subtracting Signed Numbers

- To subtract a negative number, add its opposite. **Or, two negatives make a positive.**

  Example; 10 - (-7) = 17

- To subtract a positive number from a negative number, add the two numbers and retain the negative sign.

  Example; -10 - 7 = -17

e. Rules for Multiplying Signed Numbers

- Multiplying numbers of **like signs**, the product (result) is **positive**

  Example; (-3)(-4) = 12
  4 x 5 = 20

Note: Two negatives make a positive in the first example

- Multiplying numbers of **unlike signs**, the result is always **negative**.

  Example; -4 x 5 = -20

- Multiplying **two or more numbers**, the product is positive if the number of negative signs is **even**.

  Example; (-2)(-3)(4) = +24

- Multiplying **two or more numbers**, the product is negative if the number of negative signs is **odd**.

  Example; (-2)(3)(4) = -24

f. Rules for Dividing Signed Numbers

- Dividing numbers of **like signs**, the quotient (result) is **positive**.

  Example; $(-6)/(-2) = +3$
  $6/2 = 3$

- Dividing numbers of **unlike signs**, the quotient is **negative**.

  Example; $-6 / 2 = -3$
  $8 / -4 = -2$

- Dividing **two or more numbers**, the quotient is positive if the number of negative signs is **even**.

  Example; $\frac{(-3)(-5)}{+5} = +3$

- Dividing **two or more numbers**, the quotient is negative if the number of negative signs is **odd**.

  Example; $\frac{(-3)(-5)(-2)}{+2} = -15$

CHAPTER TWO

THE DECIMAL SYSTEM

Student Objective - Students will be able to explain the basis of our numbering system, properly write, read and verbalize numbers and understand the expanded notation of our numbering system.

Student Benefits - Assessors deal with large numbers every day and must be able to properly convey numbers verbally and in written form to the many users of their data and files. The ability to correctly convey numbers to the public will inspire confidence in the assessment official.

1. Base 10 Foundation

- A numbering system consists of a set of symbols representing numbers.
- Our numbering system is based on ten symbols.
- These ten symbols are; 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.
- The 10 symbols are commonly called digits or numerals
- Any conceivable number is possible by arranging the 10 digits.
- This is known as a decimal system.

2. Place Value

- Place value facilitates naming and writing numbers.
- Equates to a number assigned to a position or location within a number.

Example; Take the numeral **465**

5 is in the units place, **6** is in the tens place, **4** is in the hundreds place

NOTE: The place value is independent of the value of the digit located in that place.

The following table represents a partial summary of the decimal system and place value.

Period	Place Value	Example Number	# of Digits
	Units	1	1
Units	Tens	10	2
	Hundreds	100	3
	Thousands	1,000	4
Thousand	Ten Thousand	10,000	5
	Hundred Thousands	100,000	6
	Millions	1,000,000	7
Million	Ten Million	10,000,000	8
	Hundred Million	100,000,000	9
Billions	Billion	1,000,000,000	10

- The value of each successive place is 10 times greater than that above it.

  Example: Ten thousand is 10 times a thousand

  A hundred million is 10 times ten million

- **Period** column denotes the name of a group of three place values normally separated by commas.

- Comma insertion facilitates reading and conveying longer numbers.

3. Expanded Notation

* Expanded notation helps understanding and communicating lengthy numbers.

* Demonstrates how numerals are put together to form larger numbers.

Rules of Expanded Notation

1. Express the value of each non zero digit separately by multiplying the digit and the place value.

2. Place plus signs between these values.

Example; 543 Expressed in expanded notation

Digit 5 in the hundreds place	=	5 x 100	= 500
Digit 4 in the tens place	=	4 x 10	= +40
Digit 3 in the units place	=	3 x 1	= + 3

The total adds to 543

Example; 15,742 expressed in expanded notation

Digit 1 in the ten thousand place	=	1 x 10,000	=	10,000
Digit 5 in the thousands place	=	5 x 1,000	=	+ 5,000
Digit 7 in the hundreds place	=	7 x 100	= +	700
Digit 4 in the tens place	=	4 x 10	= +	40
Digit 2 in the units place	=	2 x 1	= +	2

The total adds to 15,742

4. Reading and Marking Numbers

* Numbers are broken down into groups of three called periods, separated by commas. (See chart above)

* Breaking numbers into groups of three facilitates reading, writing, and interpreting lengthy numbers.

* Do <u>not</u> use the word **and** when reading whole numbers to separate the place values or periods.

Example; 1,062 is properly read, "one thousand, sixty-two"
It is **not read** "one thousand and sixty-two"

* The singular is used when reading the period names.

Example; 123,456 is read "one hundred twenty-three thousand (**not thousands**), four hundred fifty-six"

* The units place or period is not named.

Example; 456 is read “four hundred, fifty-six” Units are not mentioned.

* Complex numbers are read by inserting commas beginning at the right and every three places thereafter moving left.

Example; 123456789 is changed to 123,456,789 and read “one hundred twenty-three million, four hundred fifty-six thousand, seven hundred eighty-nine”

CHAPTER THREE

FUNDAMENTALS OF ARITHMETIC

Student Objective - Students will practice adding, subtracting, multiplying and dividing without the use of a calculator. Additionally, general rules and principles of each of the basic arithmetic operations will be reviewed.

Student Benefits - Many problems in mathematics and in the assessment office are word or story problems which can be easily converted into mathematical notation and equations. In those forms problems are easily solved. Familiarity with the rules and principles of the four arithmetic operations will allow for easier problem solving and ensure correct answers.

1. Addition

Rule: The sum of zero and any whole number is that whole number.

Example; $A + 0 = A$, or $7 + 0 = 7$

a. <u>Commutative Law of Addition</u> - The order in which two whole numbers are added does not affect the outcome.

Example; $a + b = b + a$, or $3 + 5 = 5 + 3$

b. <u>Associative Law of Addition</u> - The way in which numbers are added in groups of two does not affect the sum.

Example; $15 + 10 + 2$ can be added in two ways,

<u>Method 1</u>	<u>Method 2</u>
$(15 + 10) + 2$	$15 + (10 + 2)$
$25 + 2$	$15 + 12$
Answer = 27	Answer = 27

Note: These laws can be used to simplify solving an addition problem.

Example;	Given 25 + 39 + 75	
This is the same as:	25 + 75 + 39	Commutative Law
Combining numbers:	100 + 39	Associative Law
Answer	139	

Example;	Given, 90 + 2 + 10 + 38	
This is the same as:	(90 + 10) + (2 + 38)	Commutative Law
Combining numbers:	100 + 40	Associative Law
Answer:	140	

c. <u>Procedures in Adding - No Carry</u>

1.) Arrange all numbers in column form aligning the digits which have the same place value.

2.) Add the digits of the units place value, then the tens place value, then the hundreds place value, etc., until all place values have been added. (Think of the expanded notation)

Example;

237 = 2 hundreds + 3 tens + 7 units
<u>552 = 5 hundreds + 5 tens + 2 units</u>
789 = 7 hundreds + 8 tens + 9 units

d. <u>Procedures in Adding - Carry Required</u>

1.) Arrange all numbers in column form aligning the digits which have the same place value.

2.) Add the columns, the units with units, the tens with tens, etc.

3.) When the sum of any column has two or more digits, bring down the right digit and carry the left digit(s) to the next higher place value.

Example; 579 = 5 hundreds + 7 tens + 9 units
215 = 2 hundreds + 1 ten + 5 units
7 hundreds + 8 tens + 14 units

14 units is the same as 1 ten and 4 units. Therefore, bring down the 4 (units) and carry the 1 ten to the left or to the tens place value.

Answer, 7 hundreds + 9 tens + 4 units

The notation can be simplified by omitting the expanded notation as follows;

$$\begin{array}{r} 1 \\ 579 \\ \underline{215} \\ 794 \end{array}$$

Remember: The order in which the addends (numbers being added) are summed does not alter the outcome. Therefore, reversing the order in which the numbers were added can serve as a check for accuracy.

2. **Subtraction**

Rule 1 - The basic rule of subtraction is M - S = D, where M is the minuend, S is the subtrahend and D is the difference.

- Check results by adding the difference and the subtrahend which should equal the minuend.

Example; 5 - 3 = 2
Check 2 + 3 = 5

Example; Subtract Down

$$\begin{array}{r} 65 \\ \underline{-\ 35} \\ 30 \end{array}$$

Add up to Check
(30 + 35 = 65)

a. <u>Subtracting Whole Numbers - No Borrowing</u>

1.) Arrange the numbers vertically with the corresponding place values of the digits in columns.

2.) Beginning in the units column and working left, subtract the digits in the respective place values.

3.) Check the result by using addition.

Example; Subtract 35 from 47

$$\begin{array}{r} 47 \\ \underline{-35} \\ 12 \end{array}$$

b. <u>Subtracting Whole Numbers - Borrowing Required</u>

1.) Arrange the numbers vertically with the corresponding place values of the numerals in columns.

2.) Beginning with the units column, and working left to the tens, then hundreds, etc.

3.) If subtraction is not possible for any place value, borrow by increasing the smaller digit by 10 and decreasing the digit to its immediate left by 1.

4.) Check with addition.

Example; Subtract 39 from 67

Step 1.

$$\begin{array}{r} 67 \\ \underline{-39} \end{array}$$

Step 2 & 3

$$\begin{array}{r} 67 \\ \underline{-39} \\ 28 \end{array}$$

Since 9 cannot be subtracted from 7, increase the units numeral by 10 to 17, reduce the numeral of the tens place by 1 (to 5) and perform the operation.

Step 4 $28 + 39 = 67$ Check by adding the difference to the subtrahend or by adding up.

3. **Multiplication**

- **Factors** are numbers which are being multiplied.
- The result of the multiplication operation is the **product.**
- Multiplication is a simplified form of addition.

Example; 3 + 3 + 3 + 3 + 3 = 15

Simplifies to; 3 x 5 = 15

a. Laws of Multiplication

1) Commutative Law - The way numbers are multiplied in groups of 2 does not change their product.

Example;	(3 x 5) x 2	=	3 x (5 x 2)
	15 x 2	=	3 x 10
	30	=	30

2) Multiplicative Identity Property - Multiplying any number by 1 results in the same number.

Example; 52 x 1 = 52 or N x 1 = N

3) Multiplicative Property of Zero - The product of any number and zero is zero.

Example; 4 x 0 = 0 or 34 * 0 = 0

Note: If a long string of numbers is being multiplied and any of the numbers is zero, the result is zero.

Example; (-345)(23) * (-435) (0) (7,678) = 0

4) Distributive Law of Multiplication - Multiplying the sum of two addends is the same as multiplying the number by each addend and adding the result.

Example;

$$3(2 + 4) = (3 \times 2) + (3 \times 4)$$
$$3(6) = 6 + 12$$
$$18 = 18$$

b. Procedures in Multiplication

1) Multiplying a number by any place value (10, 100, 1,000, etc.)

a.) Omit the zeros to the right of the first non zero digit and multiply the remaining two numbers.

b.) Attach the same number of omitted zeros to that product.

Example; Multiply 6 times 6,000

Step 1.	Omit the zeros.	6 x 6 = 36
Step 2.	Attach the zeros	36,000 = The answer

2) Basic Multiplication - No Carry

a.) Arrange the factors vertically, aligning the units digits.

b.) Beginning at the right and working in order to the left, multiply each digit of the two factors recording the result below the product line.

Example; Multiply 931 and 3

Step 1. Arrange factors

$$\begin{array}{r} 931 \\ \underline{3} \end{array}$$

Step 2.

$$2{,}793$$

Order of calculation;

3 x 1 = 3
3 x 3 = 9
3 x 9 = 27

3) <u>Basic Multiplication - Carry Needed</u>

a.) Arrange the factors vertically, aligning the units digits.

b.) Beginning at the right and working in order to the left, multiply each digit of the two factors. If the product has two digits, bring down the units digit and carry the tens digit to the preceding (next left) column.

c.) Multiply the next two digits and **add** the amount carried over. Repeat step "c" until the problem has been completed.

Example; Multiply 924 by 3

Step 1. Align factors

```
 9 2 4
   x 3
```

Step 2.

A. Multiply 4 and 3 and carry 1 into the tens column.
B. Multiply 3 and 2, add 1
C. Multiply 3 and 9

```
   1
 9 2 4
     3
2,772
```

4) <u>Multiplying Numbers of Any Size</u>

a.) Arrange the factors vertically, aligning the units digits.

b.) Beginning at the right and proceeding in order to the left, multiply each digit of the two factors. Remember to carry where appropriate.

c.) Align the product with the digit of the multiplier.

d.) Add the products.

Example; Multiply 48 by 35

Step 1.	Align factors	48	
		x 35	
Step 2.	Multiply	240	(5 x 48 = 240)
	Multiply	144	(3 x 48 = 144)
Step 3.	Add	1,680	

Note: Since the 3 in the multiplier 35 is in the tens place, remember to shift one place to the left when placing the 4 from multiplying 3 times 8 yielding 24. Carry the 2 and add it to the product of 3 times 4 to attain the 14. The calculation is actually 48 times 30 but the zero is dropped for simplicity. If not dropped the zero would align with the units place.

4. **Division**

- **Dividend** - The number being divided.
- **Divisor** - The number being divided into the dividend.
- **Quotient** - The result or answer to the division problem.

- **Rule** - The basic division formula is: Dividend / Divisor = Quotient

  Example; 20/2 = 10

- **Rule** - The check for division is multiplication, Dividend = Divisor x Quotient

  Example; 20 = 2 x 10

a. <u>Division Identity</u> - Dividing any number by 1 results in the same number.

   Example; 6/1 = 6 or, 23 ÷ 1 = 23

b. <u>Dividing Zero</u> - Dividing zero by any non zero number results in a quotient which is zero.

   Example; 0 ÷ 6 = 0 or, 0 ÷ 23 = 0

c. <u>Dividing by Zero</u> - Dividing any non zero number by zero results in a meaningless quotient.

   Example; 6 ÷ 0 = Meaningless quotient

d. <u>Long Division - 4 Step Process</u>

   1.) Divide
   2.) Multiply
   3.) Subtract
   4.) Bring down the next dividend digit
   5.) Repeat as necessary

Note: As long as a digit remains in the dividend, another complete 4 step cycle must be completed.

Example; Divide 736 by 16

Step 1. Divide - Looking left to right at the dividend, identify the first digit or digits into which at least 1 whole divisor can be divided, (73). Divide 73 by 16, place 4 over the dividend.

```
   4
16) 736
```

Step 2. Multiply - Four times 16, the product is 64. Place under the 73.

```
   4
16) 736
    64
```

Step 3. Subtract - 64 from 73. The remainder is 9 and it is placed under the units digit of the subtraction operation.

```
   4
16) 736
    64
     9
```

Step 4. Bring down next digit of dividend - 6 is brought down and placed next to the remainder 9.

```
   4
16) 736
    64
     96
```

Step 5. Repeat as necessary - Repeat the sequence, finding that 46 is the quotient in this problem.

```
   46
16) 736
    64
     96
     96
      0
```

Note: If the number is not evenly divisible, place a decimal point after the last number in the quotient and bring down zeros repeating the steps above for as many places as desired.

Note: Checking Division - as discussed earlier, multiply the quotient by the divisor.

Example; 46 x 16 = 736

e. Division of Numbers Ending in Zero - When dividing by 10, 100, 1000, etc., simply move the decimal point of the dividend to the left the same number of places as there are zeros in the divisor.

Example;
246 ÷ 10 = 24.6
246 ÷ 100 = 2.46
246 ÷ 1,000 = .246
246 ÷ 10,000 = .0246
246,000 ÷ 1,000 = 246